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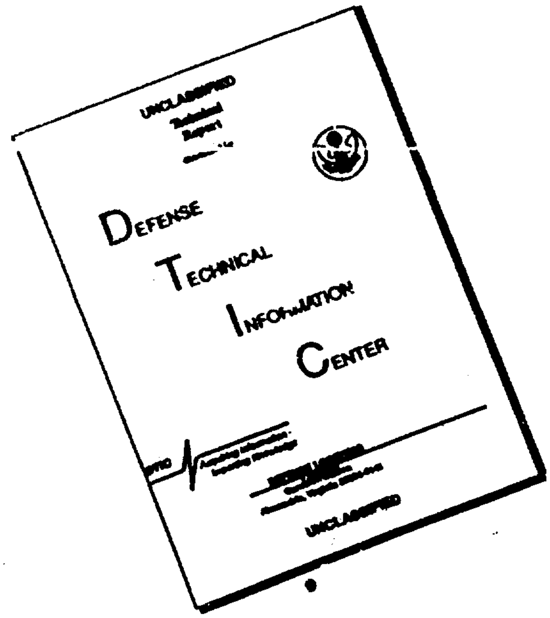

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Morris D. Friedman
Russian Translation

Three Dimensional Diffraction Problem for Electro-
Magnetic Oscillations

D. Z. Avazashvili

Soobshchenie, A. N. Gruz. SSR, vol. 14, No.6, 1953
pp. 321-328

In this paper I consider the problem of diffraction in a three-dimensional space, following the basic method used by V. D. Kupradze to solve the plane problem of the diffraction of electromagnetic waves [1,2].

§ 1. In an infinite space with electromagnetic constants $\varepsilon_0, \mu_0, \sigma_0$ let there be n successive non-intersecting enclosures bounded by the regular surfaces (see [1]) S_ν ($\nu = 1,2,\ldots,n$). The electromagnetic constants of the media occupying the successive enclosures - the dielectric constant, magnetic permeability and conductivity coefficient - we denote, respectively, by $\varepsilon_\nu, \mu_\nu, \sigma_\nu$. The region bounded by S_ν (assuming no subsequent enclosure) we denote by T_ν, the outer boundary of the surface by S_1 and the outer infinite region by T_0; the region included between S_ν and $S_{\nu+1}$ by $T_\nu - T_{\nu+1}$. Here, let $T_{0,\nu} \equiv T_0$ and $T_{n,n+1} \equiv T_n$. Moreover, let

$$k^2(M) = \begin{cases} k_0^2, & M \subset T_0 \\ k^2, & M \subset T_{\nu,\nu+1} \\ \frac{1}{2}(k_\nu^2 + k_{\nu+1}^2), & M \subset S_\nu \end{cases}$$

where

$$k_j^2 = \frac{\omega^2 \varepsilon_j \mu_j + 4\pi i \omega \sigma_j \mu_j}{c^2}; \qquad \operatorname{Im} k_j \geq 0 \quad (j = 0,1,2,\ldots,n)$$

The complex vectors of the electric and magnetic electromagnetic field intensity are $\vec{E}$ and $\vec{H}$, respectively.

The problem is formulated as follows:

Required to find $\vec{E}$ and $\vec{H}$ satisfying the conditions (see [3]):

$$
(1.1)\begin{cases}
1.\ \ \mathrm{rot}\,\vec{H} = \dfrac{4\pi\sigma_j - i\omega\varepsilon_j}{c}\,\vec{E} + \dfrac{4\pi}{c}\,\vec{G}_0 & 2.\ \ \mathrm{rot}\,\vec{E} = \dfrac{i\omega\mu_j}{c}\,\vec{H} \\
3.\ \ \mathrm{div}\,\vec{E} = 4\pi\rho_0 & 4.\ \ \mathrm{div}\,\vec{H} = 0 \ \text{ in } \ T_{j,j+1} \\
5.\ \ (E_s)_\nu = (E_s)_{\nu-1} & 6.\ \ (H_s) = (H_s)_{-1} \ \text{ on } \ S_\nu \\
7.\ \ \vec{E} = \exp(ik_\circ r)\,O(1/r) \ \ ; \ \ \dfrac{\partial\vec{E}}{\partial r} - ik_\circ\vec{E} = \exp(ik_\circ r)\,o(1/r) & \\
8.\ \ \vec{H} = \exp(ik_\circ r)\,O(1/r) \ \ ; \ \ \dfrac{\partial\vec{H}}{\partial r} - ik_\circ\vec{H} = \exp(ik_\circ r)\,o(1/r) \ \text{at infinity}^1. &
\end{cases}
$$

where

$$
\vec{G}_0(M) = \begin{cases} \vec{G} & M \subset T_\circ \\ 0 & M \subset T_{\nu,\nu+1} \end{cases}
$$

$\vec{G}$ is a given vector characterizing a source which is continuously differentiable to the second order inclusively:

$$
\rho_0(M) = \begin{cases} \dfrac{1}{\varepsilon_\circ}\,\rho & M \subset T_\circ \\ 0 & M \subset T_{\nu,\nu+1} \end{cases}
$$

ρ is the electric volume-charge density, also a given and continuously-differentiable function; ω and c are the oscillation frequency and the velocity of light in a vacuum; $(E_s)_\nu$, $(H_s)_\nu$ and $(E_s)_{\nu-1}$ $(H_s)_{\nu-1}$ respectively, are the limit values of the tangential components of $\vec{E}$ and $\vec{H}$ within and without the surface S_ν ; r is a radius-vector; $r\,o(1/r) \to 0$ as $r \to \infty$; $r\,O(1/r)$ is bounded as $r \to \infty$.

§ 2 . By virtue of (1.1_4), the vector $\vec{H}$ in $T_{j,j+1}$ $(j = 0,1,2,\ldots,n)$ will be sought as

$$
(2.1) \qquad \vec{H} = \frac{1}{\mu_j}\,\mathrm{rot}\,\vec{F}
$$

---

1. When $k_\circ$ is a real constant (1.1_7) and (1.1_8) become:

$$
\vec{E} = O(1/r) \ ; \ \frac{\partial\vec{E}}{\partial r} - ik_\circ\vec{E} = o(1/r) \ ; \ \vec{H} = O(1/r) \ ; \ \frac{\partial\vec{H}}{\partial r} - ik_\circ\vec{H} = o(1/r) \ .
$$

where $\vec{F}$ is the vector field potential. Using (2.1) in (1.1_2) we obtain:

$$\vec{E} = \text{grad}\,\varphi + \frac{i\omega}{c}\vec{F} \qquad \text{in } T_{j,j+1} \tag{2.2}$$

where φ is the scalar field potential. The vector $\vec{F}$, introduced in (2.1), in determined with the accuracy of a component and is the gradient of an arbitrary function and, obviously, the potential φ is also not uniquely defined. To eliminate this indefiniteness, let us require that this condition be fulfilled (in the $T_{j,j+1}$ region)

$$\text{div}\,\vec{F} - \frac{\mu_j(4\pi\sigma_j - i\varepsilon_j\omega)}{c}\varphi = 0 \quad \text{or} \quad \text{div}\,\vec{F} = \frac{c}{i\omega}k_j^2\varphi \tag{2.3}$$

Let us put $\vec{H}$ and $\vec{E}$ from (2.1) and (2.2) into (1.1_1) and let us use (2.3); we obtain

$$\Delta\vec{F} + k_0^2\vec{F} = \frac{-4\pi\mu_0}{c}\vec{G} \qquad \text{in } T. \tag{2.4}$$

$$\Delta\vec{F} + k^2\vec{F} = 0 \qquad \text{in } T_{\nu,\nu+1}\ ;\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2.5}$$

By virtue of (2.3) and (2.2) we obtain from (1.1_3)

$$\Delta\varphi + k_0^2\varphi = \frac{4\pi}{\varepsilon_0} \qquad \text{in } T. \tag{2.6}$$

$$\Delta\varphi + k^2\varphi = 0 \qquad \text{in } T_{\nu,\nu+1} \tag{2.7}$$

Moreover, from (1.1_1) it is evident that in T.

$$\text{div}\,\vec{E} = \frac{-4\pi}{4\pi\sigma_0 - i\varepsilon_0\omega}\,\text{div}\,\vec{G} \tag{2.8}$$

Now from (1.1_3) and (2.8) there results

$$\rho = \frac{-\varepsilon_0}{4\pi\sigma_0 - i\varepsilon_0\omega}\,\text{div}\,\vec{G} \tag{2.9}$$

Let us note that (2.6) and (2.7) are consequences of (2.4) and (2.5). In order to confirm this it is sufficient to take the divergence of (2.4)

and (2.5) and to use (2.3) and (2.9).

Let us put $\mu_j = 1$ $(j = 0, 1, 2, \ldots, n)$, then in place of the boundary conditions (1.1_5) and (1.1_6) we will have the following:

(2.10) 1. $(E_s)_\nu = (E_s)_{\nu-1}$; 2. $(H)_\nu = (H)_{\nu-1}$

By virtue of (2.1), (2.10_2) is fulfilled if $\vec{F}$ satisfies

$$(\text{rot}\,\vec{F})_\nu = (\text{rot}\,\vec{F})_{\nu-1}$$

By virtue of (2.2), (2.10_1) is fulfilled if we have on S_ν :

$$\left(\frac{\partial\varphi}{\partial s} + \frac{i\omega}{c}F_s\right)_\nu = \left(\frac{\partial\varphi}{\partial s} + \frac{i\omega}{c}F_s\right)_{\nu-1}$$

Evidently, the latter always occurs if these boundary conditions are fulfilled on S_ν :

$$\left(\frac{\partial\varphi}{\partial s}\right)_\nu = \left(\frac{\partial\varphi}{\partial s}\right)_{\nu-1}; \qquad (F_s)_\nu = (F_s)_{\nu-1}$$

Finally, the diffraction problem reduces to two boundary problems for the oscillation equations.

To find $\vec{F}$ requires solving the boundary problem:

(2.11)
1. $\Delta\vec{F} + k_0^2\vec{F} = \frac{-4\pi}{c}\vec{G}$ in T_0
2. $\Delta\vec{F} + k_\nu^2\vec{F} = 0$ in $T_{\nu,\nu+1}$
3. $(\text{rot}\,\vec{F})_\nu = (\text{rot}\,\vec{F})_{\nu-1}$; $(F_s)_\nu = (F_s)_{\nu-1}$ in S_ν
4. $\vec{F} = \exp(ik_0r)\,o(1/r)$; $\frac{\partial\vec{F}}{\partial r} - ik_0\vec{F} = \exp(ik_0r)\,o(1/r)$ at infinity.

To find φ , the problem is solved

(2.12)
1. $\Delta\varphi + k_0^2\varphi = \frac{4\pi}{\varepsilon_0}\rho$ in T_0
2. $\Delta\varphi + k_\nu^2\varphi = 0$ in $T_{\nu,\nu+1}$
3. $\left(\frac{\partial\varphi}{\partial s}\right)_\nu = \left(\frac{\partial\varphi}{\partial s}\right)_{\nu-1}$ on S_ν
4. $\varphi = \exp(ik_0r)\,o(1/r)$; $\frac{\partial\varphi}{\partial r} - ik_0\varphi = e^{ik_0r}\,o(1/r)$ at infinity.

Here $\vec{F}$ and φ, found from (2.11) and (2.12), must satisfy condition (2.3).

§ 3. The solutions of boundary problems (2.11) and (2.12), respectively, are expressed through solutions of the following integral equations:

$$(3.1)\qquad \vec{F}(M) = \frac{1}{4\pi}\sum_{j=0}^{n-1}\left\{(k_{j+1}^2 - k_j^2)\int_{T_{j+1}} \vec{F}(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N\right\} + \vec{f}(M)$$

where

$$\vec{f}(M) = \frac{1}{c}\int_{T_0} \vec{G}(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N$$

$$(3.2)\qquad \frac{c}{i\omega}k^2(M)\varphi(M) = \frac{c}{4\pi i\omega}\sum_{j=0}^{n-1}(k_{j+1}^2 - k_j^2)\left\{k^{2\prime}\int_{T_{j+1}}\varphi(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N\right.$$

$$\left. + \int_{S_{j+1}}\varphi(N)\,\frac{\partial}{\partial n_N}\left(\frac{e^{ik_0 r(M,N)}}{r(M,N)}\right)ds_N\right\} + L(M)$$

$$L(M) = \sum_{j=0}^{n-1}\left\{\frac{k_{j+1}^2 - k_j^2}{4\pi}\int_{S_{j+1}} F_n(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,ds_N - \frac{1}{c}\int_{S_{j+1}} G_n(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,ds_N\right\}$$

$$- \frac{4\pi\sigma_0 - i\omega\varepsilon_0}{c\varepsilon_0}\int_{T_0}\rho(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N$$

$$k^{2\prime} = k^2(M)\,; \qquad M \subset T_{\nu,\nu+1} \quad (\nu = 1,2,\ldots,n)$$

F_n and G_n are the projections of $\vec{F}$ and $\vec{G}$ on the interior normal.

The volume integrals in (3.1) and (3.2), taken over the infinite region T_0, exist since $\vec{G}$ and $\vec{\rho}$ are bounded and $\operatorname{Im} k_0 > 0$. For real k_0, $\vec{G}$ and $\vec{\rho}$ must satisfy some existence condition of the integrals over T_0.

(3.1) and (3.2) represent, respectively, the ordinary and loaded Fredholm integral equation of the second kind (as is known, Fredholm theory applies to the latter).

These equations are completely analogous to the equations of V. D. Kupradze which were constructed in [1,2] for electric and magnetic vectors.

The integral equations (3.1) and (3.2) were studied completely also, as was done by V. D. Kupradze (see [1] ch. 3), for the plane diffraction problem. Condition (2,3) remains to be satisfied.

Let us introduce the vector

$$\operatorname{grad} \chi = \frac{c}{4\pi i\omega} \sum_{j=0}^{n-1} (k_j^2 - k_{j+1}^2) \int_{S_{j+1}} \varphi(N)\, \vec{n}(N)\, \frac{e^{ik_o r(M,N)}}{r(M,N)}\, ds_N$$

where $\vec{n}(N)$ is the direction of the interior normal at the point $N \subset S_{j+1}$, $\varphi(N)$ is the solution of (3.2), and we form the vector

$$\vec{F}_1(M) = \vec{F}(M) + \operatorname{grad} \chi \tag{3.3}$$

The vector (3.3), obviously, satisfies (2.11), hence we have from (3.3):

$$\begin{aligned}
\operatorname{div} \vec{F}_1(M) = \frac{1}{4\pi} \sum_{j=0}^{n-1} (k_{j+1}^2 - k_j^2) \Bigg\{ \int_{T_{j+1}} \operatorname{div} \vec{F}(N)\, \frac{e^{ik_o r(M,N)}}{r(M,N)}\, d\tau_N \\
+ \frac{c}{i\omega} \int_{S_{j+1}} \varphi(N) \frac{\partial}{\partial N} \frac{e^{ik_o r(M,N)}}{r(M,N)}\, ds_N \Bigg\} \\
+ \sum_{j=0}^{n-1} \Bigg\{ \frac{k_{j+1}^2 - k_j^2}{4\pi} \int_{S_{j+1}} F_n(N)\, \frac{e^{ik_o r(M,N)}}{r(M,N)}\, ds_N \\
- \frac{1}{c} \int_{S_{j+1}} G_n(N)\, \frac{e^{ik_o r(M,N)}}{r(M,N)}\, ds_N \Bigg\} \\
- \frac{4\pi\sigma_o - i\omega\varepsilon_o}{c\varepsilon_o} \int_{T_o} \vec{\rho}(N)\, \frac{e^{ik_o r(M,N)}}{r(M,N)}\, ds_N
\end{aligned} \tag{3.4}$$

Subtracting (3.2) from (3.4), we obtain:

$$\begin{aligned}
\operatorname{div} \vec{F}_1(M) - \frac{c}{i\omega} k^2(M) \varphi(M) = \frac{1}{4\pi} \sum_{j=0}^{n-1} (k_{j+1}^2 - k_j^2) \int_{T_{j+1}} \left[\operatorname{div} \vec{F} - \frac{c}{i\omega} k^{2\prime} \varphi(N)\right] \times \\
\times \frac{e^{ik_o r(M,N)}}{r(M,N)}\, d\tau_N
\end{aligned} \tag{3.5}$$

If in particular, we fulfill the condition:

$$\int_{T_{j+1}} \frac{e^{ik_0 r(M,N)}}{r(M,N)} \frac{\partial}{\partial n}\left(\frac{e^{ik_0 r(M,N)}}{r(M,N)}\right)\cos(r,n_{N_1})d\tau_N = 0 \quad (j=0,1,\dots,n-1)$$

where $N \subset S_{j+1}$ then (3.5) becomes

$$\operatorname{div}\vec{F}_1(M) - \frac{c}{i\omega}k^2(M)\varphi(M) = \frac{1}{4\pi}\sum_{j=0}^{n-1}(k_{j+1}^2 - k_j^2)\int\left[\operatorname{div}\vec{F}_1(N) - \frac{c}{i\omega}k^2\varphi(N)\right]\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N$$

From which follows (see [1]):

$$\operatorname{div}\vec{F}_1(M) - \frac{c}{i\omega}k^2(M)\varphi(M) = 0 \quad \text{or} \quad \operatorname{div}\vec{F}_1(M) = \frac{c}{i\omega}k^2(M)\varphi(M)$$

i.e., (2.3) .

In the general case, we consider the system:

$$\vec{F}(M) = \frac{1}{4\pi}\sum_{j=1}^{n-1}\left\{(k_{j+1}^2 - k_j^2)\int_{T_{j+1}}\vec{F}(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N + \frac{c}{i\omega}(k_j^2 - k_{j+1}^2)\int_{S_{j+1}}\varphi(N)\vec{n}(N) \times \right.$$
$$\left. \times \frac{e^{ik_0 r(M,N)}}{r(M,N)}ds_N\right\} + \vec{f}(M)$$

(3.6)

$$\frac{c}{i\omega}k^2(M)\varphi(M) = \frac{c}{4\pi i\omega}\sum_{j=0}^{n-1}(k_{j+1}^2 - k_j^2)\left\{k^{2\prime}\int_{T_{j+1}}\varphi(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N\right.$$
$$\left. + \int_{S_{j+1}}\varphi(N)\frac{\partial}{\partial n_N}\left(\frac{e^{ik_0 r(M,N)}}{r(M,N)}\right)ds_N + \frac{i\omega}{c}\int_{S_{j+1}}F_n(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}ds_N\right\}$$
$$+ L(M)$$

where

$$\vec{f}(M) = \frac{1}{c}\int_{T_0} G(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N$$

$$L(M) = \frac{-1}{c}\sum_{j=0}^{n-1}\int_{S_{j+1}} G_n(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}ds_N - \frac{4\pi\sigma_0 - i\omega\varepsilon_0}{c\varepsilon_0}\int_{T_0}\rho(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N$$

The functions $\vec{F}$ and φ, determined from (3.6), satisfy (2.11), (2.12) and (2.3). Therefore, (3.6) and (1.1) are mutually equivalent. In particular, the homogeneous problem (1.1.) is equivalent to the corresponding homogeneous system of integral equations (3.6.).

§ 4. Let us study the system (3.6). For simplicity, let us consider the case $n = 1$:

$$\vec{F}(M) = \frac{(k_1^2 - k_0^2)}{4\pi}\int_{T_1}\vec{F}(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N + \frac{c(k_0^2 - k_1^2)}{4\pi i\omega}\int_{S_1}\varphi(N)\vec{n}(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,ds_N + \vec{f}(M)$$

(4.1)
$$\frac{c}{i\omega}k^2(M)\varphi(M) = \frac{ck_1^2(k_1^2 - k_0^2)}{4\pi i\omega}\int_{T_1}\varphi(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,d\tau_N + \frac{c(k_1^2 - k_0^2)}{4\pi i\omega}\int_{S_1}\varphi(N)\frac{\partial}{\partial n_N}\left(\frac{e^{ik_0 r(M,N)}}{r(M,N)}\right)ds_N + \frac{k_1^2 - k_0^2}{4\pi}\int_{S_1}F_n(N)\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,ds_N + L(M)$$

Let $M \subset T_1$; let us introduce the notation:

$\Phi_1(M) = F_x(M)$; $\Phi_2(M) = F_y(M)$; $\Phi_3(M) = F_z(M)$; $\Phi_4(M) = \varphi(M)$

$\Psi_1(M) = f_x(M)$; $\Psi_2(M) = f_y(M)$; $\Psi_3(M) = f_z(M)$; $\Psi_4(M) = L(M)$

$$\lambda = \frac{k_1^2 - k_0^2}{4\pi}\ ;\quad A_{\alpha,\beta}(M,N) = \begin{cases} -\dfrac{\exp ik_0 r(M,N)}{r(M,N)} & \text{for } \alpha = \beta \\ 0 & \text{for } \alpha \neq \beta\ (\alpha,\beta = 1,2,3,4) \end{cases}$$

$$B_{11}(M,N) = B_{12}(M,N) = B_{13}(M,N) = 0\ ;\quad B_{14}(M,N) = \frac{c\cos(n,\xi)}{i\omega}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}$$

$$B_{21}(M,N) = B_{22}(M,N) = B_{23}(M,N) = 0\ ;\quad B_{24}(M,N) = \frac{c\cos(n,\eta)}{i\omega}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}$$

$$B_{31}(M,N) = B_{32}(M,N) = B_{33}(M,N) = 0\ ;\quad B_{34}(M,N) = \frac{c\cos(n,\zeta)}{i\omega}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}$$

$$B_{41}(M,N) = -\frac{ik\cos(n,\xi)}{ck_1^2}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,; \qquad B_{42}(M,N) = -\frac{ik\cos(n,\eta)}{ck_1^2}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}$$

$$B_{43}(M,N) = -\frac{ik\cos(n,\zeta)}{ck_1^2}\,\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,; \qquad B_{44}(M,N) = -\frac{1}{k_1^2}\,\frac{\partial}{\partial n_N}\left(\frac{e^{ik_0 r(M,N)}}{r(M,N)}\right)$$

In the sequel we will denote the set of functions $\Phi_1, \Phi_2, \Phi_3, \Phi_4$ by the vector $\vec{\Phi}(\Phi_1, \Phi_2, \Phi_3, \Phi_4)$.

Similarly, $\vec{\Psi}(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ is a vector with components Ψ_1, Ψ_2, Ψ_3, Ψ_4. Let $A(M,N)$ be the matrix

$$A(M,N) = \|A_{\alpha,\beta}(M,N)\| = \begin{Vmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{Vmatrix}$$

and $B(M,N)$

$$B(M,N) = \|B_{\alpha,\beta}(M,N)\| = \begin{Vmatrix} 0 & 0 & 0 & B_{14} \\ 0 & 0 & 0 & B_{24} \\ 0 & 0 & 0 & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{Vmatrix}$$

Then (4.1) can be written

$$(4.2) \quad \vec{\Phi}(M) + \lambda \int_{T_1} A(M,N)\,\vec{\Phi}(N)\,d\tau_N + \lambda \int_{S_1} B(M,N)\,\vec{\Phi}(N)\,ds_N = \vec{\Psi}(M)$$

Equation (4.2) is a loaded Fredholm equation of the second kind. This can be written in the usual form if we introduce a new kernel and new differential.

Let us put $(M \subset T_1 + S_1)$

$$K(M,N) = \begin{cases} A(M,N) & \text{if } N \subset T_1 \\ B(M,N) & \text{if } N \subset S_1 \end{cases} \qquad d\omega_N = \begin{cases} d\tau_N & \text{in } T_1 \\ ds_N & \text{on } S_1 \end{cases}$$

Then (4.2) becomes

$$(4.3) \qquad \vec{\Phi}(M) + \lambda \int_{T_1+S_1} K(M,N)\,\vec{\Phi}(N)\,d\sigma_N = \vec{\Psi}(M)$$

As is known, Fredholm theory is applicable to (4.3) (see V. I. Smirnov [4]).

The proof of the uniqueness theorem for (1.1) is given in [5]. Therefore, by virtue of the equivalence, the homogeneous system (4.3₀):

$$(4.3_0) \qquad \vec{\Phi}(M) + \lambda \int_{T_1+S_1} K(M,N)\,\vec{\Phi}(N)\,d\sigma_N = 0$$

has only a trivial solution. This means that (4.1) is solvable for any right side and the existence theorem is proved.

Tiflis Inst of RR Eng. July, 1953

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